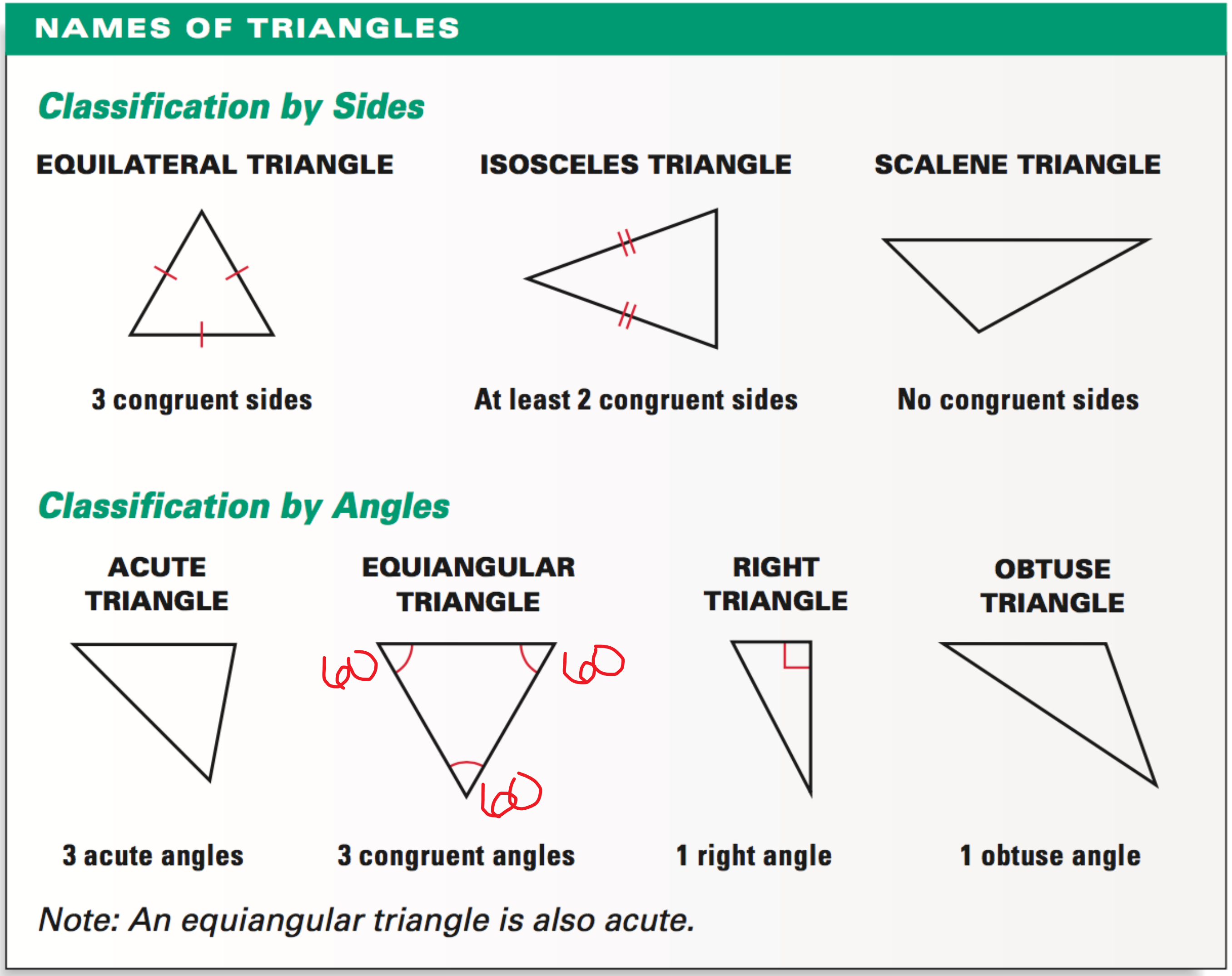
Chapter 4

Congruent Triangles

Section 1
Triangles and Angles

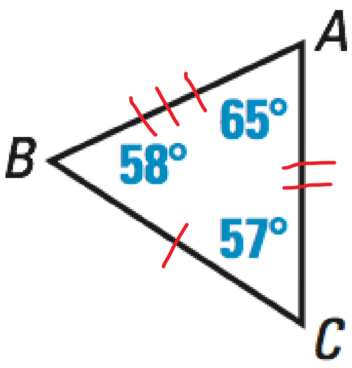
GOAL 1: Classifying Triangles

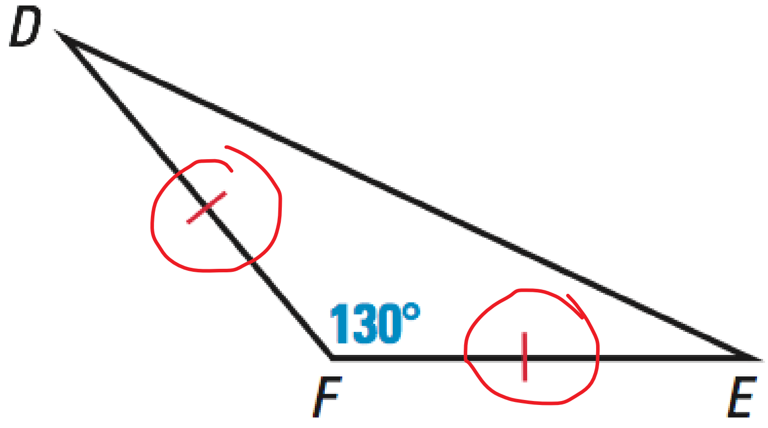
A triangle is a figure formed by three segments joining three noncollinear points. A triangle can be classified by its sides and by its angles, as shown in the definitions below.



Example 1: Classifying Triangles

*When classifying a triangle, you need to be as specific as possible!



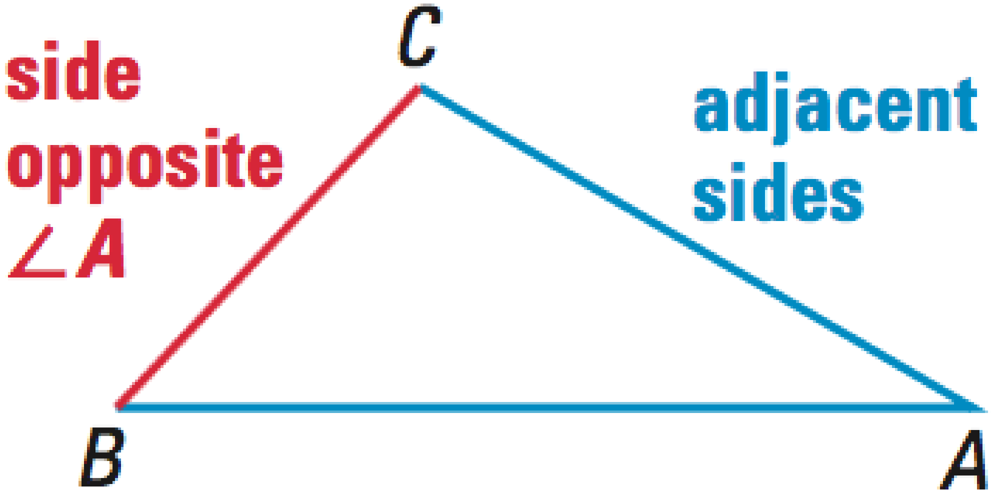


scalene, acute

isosceles, obtuse

Each of the three points joining the sides of a triangle is a _______vertex_______. For example, in ΔABC, points A, B, and C are vertices.

In a triangle, two sides sharing a common vertex are _______________adjacent sides________________. In ΔABC, $\overline{CA}$ and $\overline{BA}$ are adjacent sides. The third side, $\overline{BC}$, is the side opposite <A.

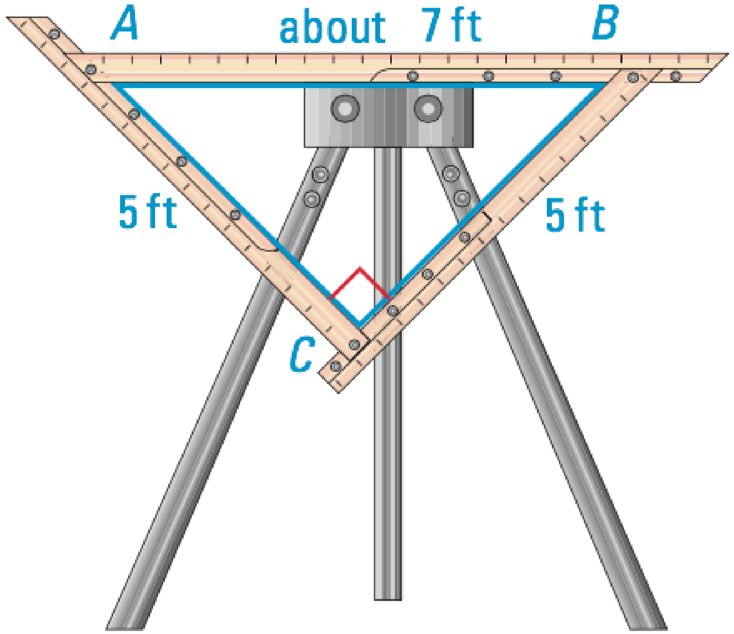


Right and Isosceles Triangles

The sides of right triangles and isosceles triangles have special names. In a right triangle, the sides that form the right angle are the _____legs_____ of the right triangle. The side opposite the right angle is the _______hypotenuse________ of the triangle.

An isosceles triangle can have three congruent sides, in which case it is equilateral. When an isosceles triangle has only two congruent sides, then these two sides are the ______legs______ of the isosceles triangle. The third side is the ___base___ of the isosceles triangle.

Example 2: Identifying Parts of an Isosceles Right Triangle



The diagram shows a triangular loom.

a) Explain why ΔABC is an isosceles triangle.

AC & BC are both 5 feet → 2 congruent sides

→ isosceles

a) Identify the legs and the hypotenuse of ΔABC. Which side is the base of the triangle?

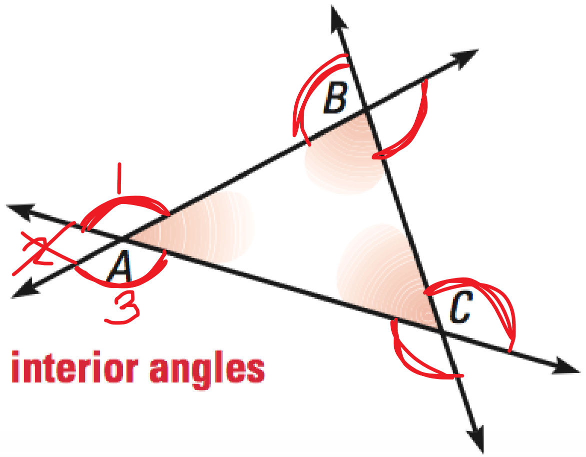
legs → CB & CA

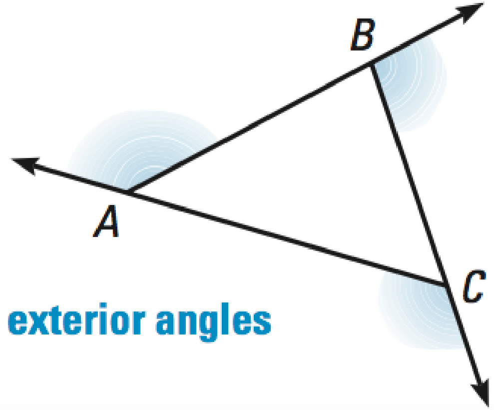
hypotenuse → AB (opposite the right angle)

base → AB (side that is different in an isosceles triangle)

GOAL 2: Using Angle Measures of Triangles

When the sides of a triangle are extended, other angles are formed. The three original angles are the ______interior angles______. The angles that are adjacent to the interior angles are the _________exterior angles____________. Each vertex has a pair of congruent exterior angles. It is common to only show one exterior angle at each vertex.



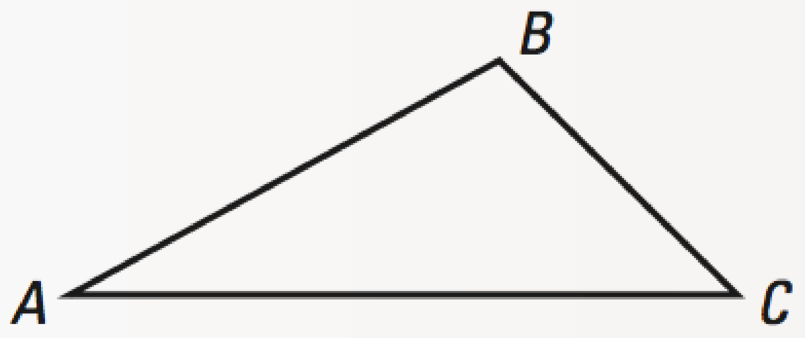


THEOREM

THEOREM 4.1 ***Triangle Sum Theorem***

The sum of the measures of the interior angles of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$

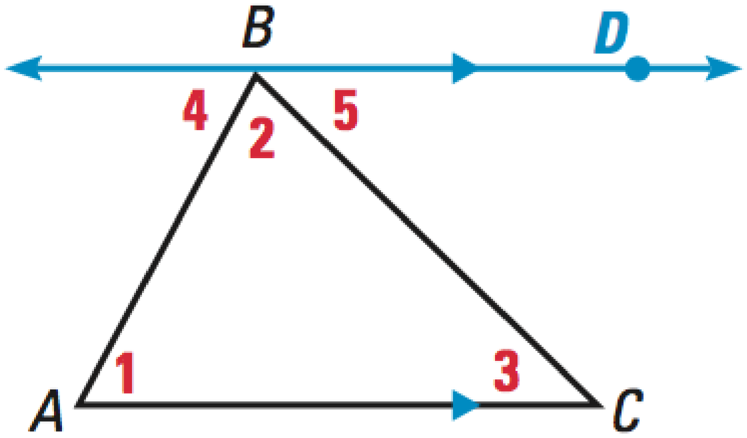


To prove some theorems, you may need to add a line, a segment, or a ray to the given diagram. Such an auxiliary line is used to prove the Triangle Sum Theorem.

GIVEN ▶ $\triangle ABC$

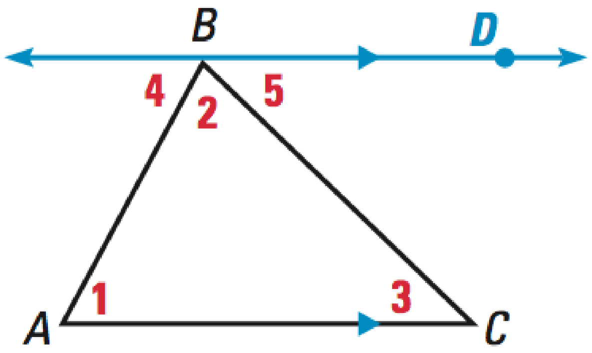
PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Plan for Proof By the Parallel Postulate, you can draw an auxiliary line through point B and parallel to $\overline{AC}$. Because $\angle 4$, $\angle 2$, and $\angle 5$ form a straight angle, the sum of their measures is $180°$. You also know that $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$ by the Alternate Interior Angles Theorem.



GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



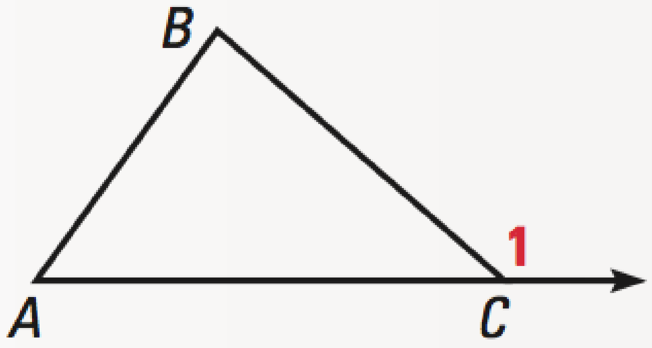
Statements	Reasons
1) Tri. ABC	1) Given
2) Draw BD || AC	2) Parallel Postulate
3) m<4 + m<2 + m<5 = 180*	3) Angle Addition Post./def. of straight <
4) <1 cong. <4; <3 cong. <5	4) Alt. Int. Angles Theorem
5) m<1 = m<4; m<3 = m<5	5) Def. of cong. <'s
6) m<1 + m<2 + m<3 = 180*	6) Substitution

THEOREM

THEOREM 4.2 ***Exterior Angle Theorem***

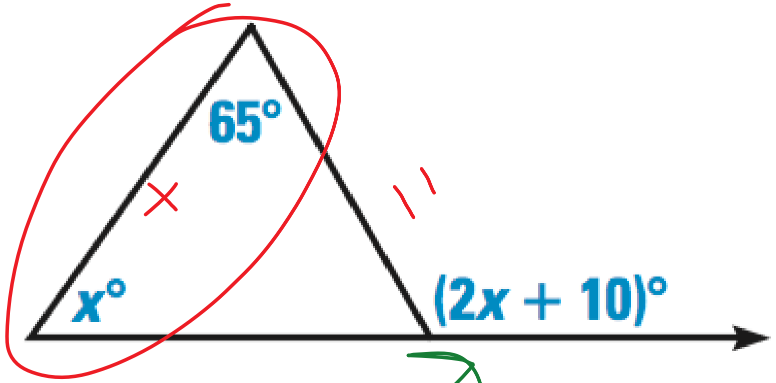
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$



Example 3: Finding an Angle Measure

You can apply the Exterior Angle Theorem to find the measure of the exterior angle shown. First write an solve an equation to find the value of x, then use the value of x to find the measure of the exterior angle.



$$\cancel{x} + 65 = 2x + \cancel{10}$$

$$-\cancel{x} \quad -10 \quad -x \quad -\cancel{10}$$

$$55 = x$$

$$2(55) + 10 = 120$$

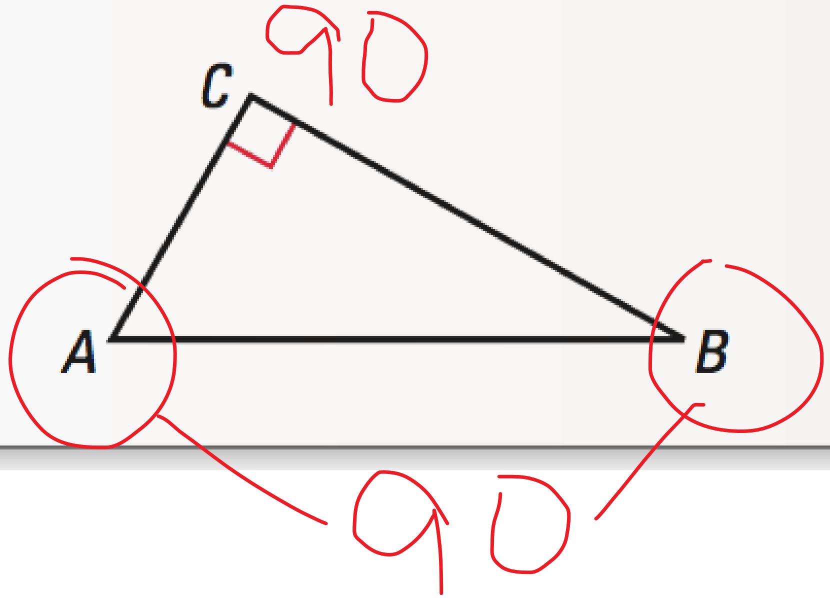
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY

COROLLARY TO THE TRIANGLE SUM THEOREM

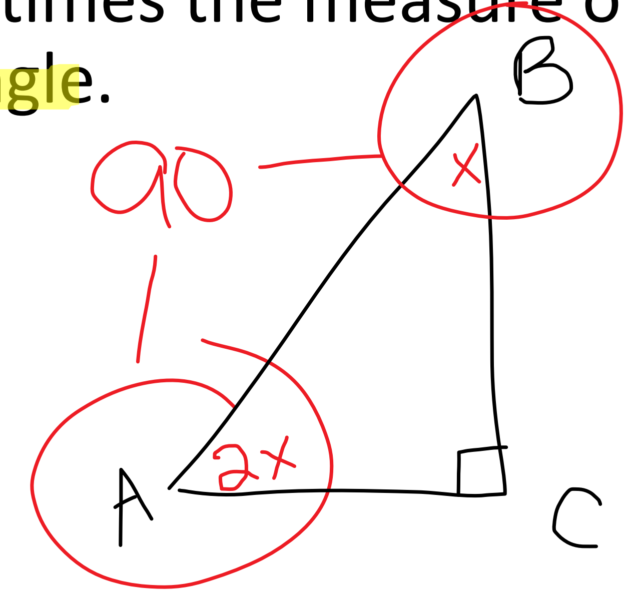
The acute angles of a right triangle are complementary.

$$m\angle A + m\angle B = 90^\circ$$



Example 4: Finding Angle Measures

The measure of one acute angle of a right triangle is two times the measure of the other acute angle. Find the measure of each acute angle.



$$x + 2x = 90$$

$$3x = 90$$

$$x = 30$$

$$\angle A \rightarrow 2(30) = 60^\circ$$

$$\angle B \rightarrow 30^\circ$$

EXIT SLIP

Finding interior angles → subtract

Finding exterior angles → add